MATH 53, Practice for Midterm 1

You should allocate 90 minutes to do the following 9 problems (starting on the back of this page). The difficulty and spread of topics are *not* indicative of the actual midterm. Most of the problems are exercises from Stewart (and I expect the actual midterm to be like that as well).

Make sure to show your reasoning, as an answer with no explanation will receive no credit on the actual exam. It is also a good habit to box your final answers.

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Problem 1 (adapted from §10.2.31)**.** Let $a, b > 0$. The equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

defines an ellipse.

a) Parametrize this ellipse so that it starts and ends at $(0, b)$, and traces the curve out once clockwise.

b) Compute the area enclosed by the ellipse.

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Problem 2 (§10.4.35)**.** Find the area inside the larger loop and outside the smaller loop of the limaçon $r = \frac{1}{2} + \cos\theta$.

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Problem 3. Consider the line L_1 given by

$$\frac{x-1}{3} = \frac{y-4}{2} = \frac{z+3}{-4}$$

and the line L_2 given by

$$\mathbf{r}(t) = \langle 4-3t, 3-2t, 4t \rangle.$$

a) Are L_1 and L_2 parallel, intersecting, or skew?

b) If they intersect, find the point at which they intersect. Otherwise, determine the distance between the two lines.

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Problem 4. Let C be the curve of intersection of the surfaces $z = \sqrt{x^2 + y^2}$ and $z = 1 - y$.

a) Parametrize the curve C.

b) Compute an equation for the tangent line to C at the point $(1, 0, 1)$.

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Problem 5.

a) Find a function $f(x, y)$ such that $f_x(x, y) = ye^{xy} + \sin y$ and $f_y(x, y) = xe^{xy} + x\cos y$.

b) (§14.3.97) On the other hand, there is no $g(x, y)$ such that $g_x(x, y) = x + 4y$ and $g_y(x, y) = 3x - y$. Explain why, *without* using integration.

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Problem 6. Let $f(x, y) = \sqrt{xy}$, and P be the point $(2, 8)$.

a) At the point P, give a unit vector pointing in the direction in which f *decreases* the most rapidly.

b) (§14.6.19) Compute the directional derivative of f at P in the direction of the point $Q = (5, 4)$.

Problem 7. The point $(2, 3, 6)$ lies on the surface S given by $z = xy$. Let H be the tangent plane to S at that point. Show that the intersection of H with S is a pair of lines, and give their equations.

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Problem 8 (§14.7.32)**.** Find the absolute maximum and minimum values of

$$f(x, y) = x + y - xy$$

on the closed triangular region with vertices $(0, 0)$, $(0, 2)$, and $(4, 0)$.

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Problem 9. Let $z = f(x, y)$, where f is a function satisfying the identity

$$f(tx, ty) = t^3 f(x, y) \tag{$*$}$$

for all t.

a) (Adapted from §14.5.55) Assume for this part that f is a very "nice" differentiable function. Show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z.$$

Hint: Differentiate $(*)$ with respect to t.

b) (Hard; do not attempt unless you are done with the rest of the exam.) Suppose that, aside from $(*)$, we only know that the domain of f is all of $\mathbb{R}^2$, and that f is continuous at all points other than $(0, 0)$. Prove that f is also continuous at $(0, 0)$. **Hint:** try switching to polar to evaluate the limit.